Assignment 3

CS 514 – Algorithms

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**Loop invariant:**  
  
At the start of each iteration of the for loop, the subarray ***A[1....i]*** is a max-heap containing the i smallest elements of ***A[1....n]*** , and the subarray ***A[i+1......n]*** contains the n-i largest elements of ***A[1....n]***, sorted.

**1. Initialization:**

The algorithm creates a max heap from the array ***A[1…….n]*** before the second **“for loop”** in the **“heapsort”** function starts. Since the sorting hasn’t begun yet, it holds that the subarray ***A[1....i]*** is a max-heap for any ***i*** at this time since the entire array is a max-heap.   
This proves that the requirement of startup has been satisfied.

### 2. ****Maintenance****:

During each iteration, the algorithm takes the root (the maximum element in the max-heap), swaps it with the element at the last position, and then reduces the heap size by one. Now, the maximum element is appropriately positioned at the array's end. The technique then uses ***`hepify’*** on the root to make sure the remaining items form a max-heap as well.

Considering the loop invariant, the subarray ***A[1....i]*** is a max-heap, and the elements in the subarray ***A[i+1....n]*** are sorted. After each iteration, the largest element of the max-heap is moved to its correct position, and ` **hepify** ` ensures that the invariant is maintained by making sure that the remaining elements are still in a max-heap structure. Hence, the maintenance condition is satisfied.

### 3. ****Termination****:

When every element has been placed in its proper location and there is just one element left in the max-heap, the loop comes to an end. We now have a sorted array since the loop invariant has been preserved throughout the process and every iteration positions the maximum member of the leftover max-heap in the proper location. Consequently, the algorithm is accurate, and the termination condition is satisfied.